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CHAPTER 13

ON THE COSTS OF RECESSIONS IN A LIFE CYCLE MODEL

Introduction

A reference to the famous Lucas's [6] calculation has appeared as the .first sentence in a number of contributions published since. Unquestionably both the counterintuitive result of Lucas, as most economists at least implicitly view fluctuations as harmful, and the simplicity of the original approach have led the profession to propose alternative methods to estimate the true costs of consumption volatility. The new methods normally dominate the original framework in terms of analytic and conceptual sophistication and, perhaps not surprisingly, normally lead to quantitative estimates of the costs of oscillations of orders of magnitude higher than the one found by Lucas. This paper adds on to the existing literature. Its primary objective is to assess the costs of cyclic shifts in economic activity to an individual. Unlike other contributions that have come after 1987 the paper invokes only very rudimentary analytical techniques and in fact the only difference between the Lucas's approach and the one presented in this paper amounts to an extension of consumer's horizon relative to the single period case analyzed by Lucas. It turns out that such a simple extension of the original model can have a profound impact on the quantitative estimates. The paper shows that the costs of .fluctuations can be as high as 10% of the average consumption to a .fifty year old consumer.

Recessions appear on average every eight years and last for about two years. Therefore, a representative consumer is expected to experience several downturns and upturns of economic activity during her professional activity. The relative high frequency of oscillations has attracted considerable attention from the profession. Consequently, analyzing both the origins of .fluctuations as well as their impact on welfare has become a major pillar of modern macroeconomics. In fact arguments, most famous that of Lucas, suggesting that there is nearly no more room for any further stabilization policy and that the profession focuses too much on an issue of little practical importance have gain recognition. However, the arguments have in fact spurred a new interest in oscillations rather than refocused the profession on more relevant topics.

The estimate of Lucas that the costs of .fluctuations are equivalent to a loss of consumption on the order of 0.0006 of the average consumption has been widely perceived as being based on .awed assumptions and too a simplistic framework. Numerous extension of the original model have been suggested since the 1980s. Economists. attempts to account for heterogeneity, Atkeson and Phelan [2], turned out to be unsuccessful. On the contrary Ball and Romer's [3] work that took into account the potential costs associated with hours of work variability revealed that the costs of fluctuations could be substantial if labor supply were relatively inelastic. Moreover, as Meltzer [8] noted the costs of .fluctuations can be indirect as investment may be higher when the economy is more stable. In addition Imrohoroglu [5] and Storesletten et al. [10] showed that the presence of idiosyncratic risk, liquidity constraints, and variability persistence could increase the original estimate significantly. The properties of CRRA utility function were also listed, Vandenbroucke [11], as potential source of a downward bias in the Lucas's result. The indirect estimates of Alvarez and Jermann [1] based on equity premium revealed that the marginal cost of consumption variability was on the order of 0.3-0.55% over a life time. In a recent paper Gali, Gertler and Lopez-Salido [4] show that cost of business cycles can be large if one takes into account the impact of output variability on the potential output. The level of development could also contribute to welfare costs of fluctuations as pointed out by Pallage and Robe [9]. Nevertheless, taking into account all the above arguments should not lead to an estimate of the costs of business cycles greater than 0.1% of the average consumption, Lucas [7].

The paper adds on to the vast .field of existing literature. However, unlike other contributions the paper shows that indeed output .fluctuations can be costly without resorting to concepts of idiosyncratic risk, market incompleteness, or heterogeneity. In fact the paper offers very little economic insight above the Lucas's framework. The sole contribution of the paper comes from the fact that the basic calculation of the welfare costs is done more carefully in a more realistic life-cycle setup. The paper assumes that individuals lives are .finite and that over the life span agents allocate their resources. At any point in time agents do decide how much to save and how much to consume conditional on their expectations of the future income streams. As soon as it is assumed that agents do not operate in a perfect foresight environment it is obvious that agents' savings always differ from the level that would exist under perfect foresight, i.e., agents either save too much or too little compared to the level in the full information case. To put it differently future income uncertainty is accounted for ex ante. Agents equalize the marginal utility in the current period with the expected marginal utility in the future. However, conditional on a given realization of the future income level the amount saved the period before is always either insufficient or excessive as agents expected the future income to be, loosely speaking, the average of the potential future realization and ex post they must deal with a specific realization that need not be equal to the average. It turns out that this under or over saving as compared to the full information case not only imposes direct costs as consumption is not smoothened out, but also indirectly as it alters the future consumption path. Assuming for illustrative purposes that a particular realization of income in a given period is below the expected value from the period before it is possible to conclude that agents must consider their level of savings to have been insufficient and must now revise the entire consumption plan downwards. Therefore, the costs not only accrue due to a shift in the level of consumption from one period to another, but also from the fact that the entire profile of consumption is changed. Alternatively, one may note that in a representative, infinite lived agent framework a given realization of uncertainty has no impact on the future consumption profile as at any point in time the number of expected future high and low realizations of income is the same whereas in a fi.xed horizon setup a given unfavorable realization can influence the consumption profile significantly as agents may simply realize that there is no enough time to recover from the unfavorable realization (the likelihood of a favorable realization that would offset the unfavorable one is miniscule. Naturally, in a stochastic rational environment the accumulation errors must on average be equal to zero and there are no .first order consequences. However, from the ex ante point of view the levels of under/over savings and consequently deviations of the level of assets from the level under perfect foresight at different points in time have a distribution, with mean zero and an expanding variance with time. Naturally, this under/over savings imposes second order cost on individuals and this cost increases with time. The paper estimates this costs and shows that in an ex ante sense the welfare cost of .fluctuations to a .fifty year old individual can be equivalent to a 10% fall in consumption.) In addition, the paper shows that if expansions are longer than recessions then the aggregate consumption exhibits fluctuations with recessions being deeper than expansions.

For illustrative purposes the exposition starts with a simple model that departs from rationality developed in section (2) then it turns in section (3) to a rational model and presents the results. The paper summarizes the findings in section (4.)

A Simple Illustrative Model

Consider an economy inhabited by a number of T cohorts. At each point in time a new cohort is born. The cohort born at a given point in time comprises a continuum of measure one of agents. The preferences of a member of the cohort as of time of birth are represented

$$U\left(\{c_t\}_{t=1}^{t=T}\right) = \sum_{t=1}^{T} \frac{c_t^{1-\theta} - 1}{1-\theta}. \quad (1)$$

by

A given cohort lives only for T periods and at any point in time there are exactly T present cohorts each of different age t where $t \in \{1,2,\ldots,T\}$. Note that the paper at this stage assumes no discounting and uses a CRRA utility function as a measure of consumption .flow valuation. The former assumption is relaxed in the due course, while the latter is kept, despite its criticism Vandenbroucke [11], to facilitate simple referential comparisons to the Lucas's [6] approach. Assume for completeness that the net real interest rate is equal to zero, i.e., r = 0 and that at each point in time each member of the cohort receives income y. In other words it is assumed that the income earned at each point in time is fixed and independent of consumer's age.

Naturally, the assumptions introduced above imply that the level of consumption is identical at all periods and simply equal to the level of income, i.e.,

$$c_t = y. \quad (2)$$

Now, let's assume that T = N*K and K = R + B, where N;K;R; and B are natural numbers. Let K be the length of the business cycle, R be the length of a recession and B be the length of an expansion. In other words, it is assumed that a given cohort lives through N full cycles during her life time. This is consistent in the qualitative sense with the fact that on average during her life time a given human being experiences six full cycles and six recessions with the average length of two years and six expansion each lasting on average eight years.

Again, let's continue to assume that the level of income is .fixed for all agents and for all cohorts at each point in time and is always equal to y. However, let's, temporarily departing from the framework of rationality, assume that all agents at time t observe their incomes, each income is equal to y, and overstate their future incomes by the amount x_t, i.e., agents expect to be receiving at each future date $y+x_t$ even though their true realized income is simply equal to y. The size of the error x_t can change over time, but at a given point in time is identical for all agents irrespective of their age or wealth. In addition, learning is ruled out at this stage, i.e., agents make no rational inferences from their past errors. Under these modified there is a natural need to index cohorts both with the current time and with their time of birth. For simplicity let's trace the evolution of consumption of an agent born at time t. In the first period of her life she receives income y, which she observes, and expects to receive $y + x_t$ at all future dates. Consequently her consumption in period t is given by

$$c_t = \frac{T-1}{T} x_t + y.$$

She enters her second period with assets, $-\frac{T-1}{T}x_t + y$, receives income y in her second period, and expects to receive y + xt+1 at all subsequent periods. Accordingly her choice of consumption in period t + 1 is simply given by

$$c_{t+1} = -\frac{x_t}{T} - \frac{x_{t+1}}{T-1} + y + x_{t+1}.$$

In general for any $i \in \{1,2,\dots,T\}$ the level of consumption can be written in a quite elegant form of, of course xt+T-1 = 0,

$$c_{t+i-1} = y + x_{t+i-1} - \sum_{j=0}^{i-1} \frac{x_{t+j}}{T-j}. \tag{3}$$

Note that irrational behavior of economic agents results in levels of consumption different from the optimal one and equal to y. However, given that the budget constraint must bind over the life time the average consumption is still equal to y per period. It is possible to assess the cost to a consumer of irrational behavior of this type. Naturally, it is fairly easy to notice that the cost differs with her age. Again, for expositional purposes let's follow the cost paid by an agent born at time t. Let ρ_i^t be the cost paid in terms of the average consumption by a consumer born at time t at time t + i - 1 (in the i'th period of her life.) One should be clear at this moment. There are at least two types of costs that can be assessed at this stage. The .first being the cost of further errors made at time t + i - 1 and beyond, i.e., it is possible to assess the cost in terms of consumption of future, from perspective of period t + i - 1; errors. Alternatively it may be worth assessing the cost in terms of consumption of both past, from the perspective of period t + i - 1, and future, from the perspective of period t + i - 1, errors, i.e., one can ask by how much would the utility as of period t + i - 1 have been higher had the agent committed no errors (past or future) in assessing her income streams. The paper follows the latter approach and evaluates the loss in terms of the average consumption due to misperception of income.

At time t the value of the realized, not expected, utility is given by

$$U_{t,t} = \sum_{i=1}^{T} \frac{\left(y + x_{t+i-1} - \sum_{j=0}^{i-1} \frac{x_{t+j}}{T-j}\right)^{1-\theta} - 1}{1-\theta}. \tag{4}$$

The level of utility that would materialize without any misperception errors is simply given by

$$U_{t,t}^{R} = \sum_{i=1}^{T} \frac{y^{1-\theta} - 1}{1-\theta} = T\frac{y^{1-\theta} - 1}{1-\theta}. \tag{5}$$

The realized level of utility 4 is smaller than the level given by 5 as consumption profile is not smoothened out optimally. The lack of smoothness imposes a cost that is equivalent to a loss of the average consumption of ρ_1^t1, where ρ_1^t 1 satisfies

$$(1-\rho_1^t)^{1-\theta} = \frac{1}{T}\sum_{i=1}^{T}\left(1+\varepsilon_{t+i-1}-\sum_{j=0}^{i-1}\frac{\varepsilon_{t+j}}{T-j}\right)^{1-\theta},$$

where $\varepsilon_{t+k-1} = \frac{x_{t+k-1}}{y}$ is the percentage error made at time t+k-1 in assessing future income streams.

Similarly, the realized utility of a consumer born at time t from the perspective of period t+k - 1 (the k'th period of life), given all decisions from periods t through t + k - 2 is given by

$$U_{t,t+k-1} = \sum_{i=k}^{T}\frac{\left(y+x_{t+i-1}-\sum_{j=0}^{i-1}\frac{x_{t+j}}{T-j}\right)^{1-\theta}-1}{1-\theta}$$

and the level of utility that would have materialize had there been no errors in assessment is given by

$$U_{t,t+k-1}^{R} = (T-k+1)\frac{y^{1-\theta}-1}{1-\theta},$$

so the cost in terms of consumption satisfies

$$(1-\rho_k^t)^{1-\theta} = \frac{1}{T-k+1}\sum_{i=k}^{T}\left(1+\varepsilon_{t+i-1}-\sum_{j=0}^{i-1}\frac{\varepsilon_{t+j}}{T-j}\right)^{1-\theta}. \quad (6)$$

Observe that both past and future errors influence the cost. Moreover, it could be even the case that the cost is actually negative. This could for example happen when on the path of consumption agents at some past dates severely under-saved. Before proceeding on to numerical estimates of the cost let's put on some economic interpretation on the results. Second order approximation of 6 allows to establish a simple expression for ρ_k^t (the percentage loss in consumption from the perspective of the k.th period of one's life).

$$\rho_k^t = \sum_{i=1}^{k-1}v_i\varepsilon_{t+i-1} + \frac{\theta}{2}\left\{\left(\sum_{i=1}^{k-1}v_i\varepsilon_{t+i-1}\right)^2 + \frac{1}{T-k+1}\sum_{i=k}^{T}(1-v_i)\varepsilon_{t+i-1}^2\right\}, \quad (7)$$

where $v_i = \frac{1}{T-i+1}$. It is apparent that the loss ρ_k^t differs significantly from an analogous expression obtained in the traditional framework. In fact there are three important differences. First of all, in the present setup the loss contains a first order term, $\sum_{i=1}^{k-1}v_i\varepsilon_{t+i-1}$. The term

disappears in expected value if errors are on average unbiased, i.e., Eε = 0. The presence of the first order term while numerically important in the current section does not constitute the essence of this paper and the ultimate results are obtained in a framework with the .first order term disappearing. Furthermore, the second order term, the expression multiplying $\frac{\theta}{2}$ contains two terms. The second term corresponding to the value in the original calculation and the .first term being novel and due to an explicit life-cycle modelling. The presence of this new term, $\left(\sum_{i=1}^{k-1} v_i \epsilon_{t+i-1}\right)^2$, forms the basis of the results of this paper. Finally, the loss is time dependent and actual consumption costs are a function of the horizon of a consumer.
To cast the issue in a more transparent setup the paper temporarily assumes that the income percentage perception errors ϵ_{t+i-1} have all mean zero and a fixed variance σ^2. In this case the value of the loss ρ_t^k in expected terms is approximately given by

$$\rho_k^t = \frac{\theta}{2}\left\{E\left(\sum_{i=1}^{k-1} v_i \varepsilon_{t+i-1}\right)^2 + \sigma^2\left(1 - \frac{\log(T-k+1)}{T-k+1}\right)\right\}.$$

More specifically if the errors are serially uncorrelated the expression can be simplified a bit further to

$$\rho_k^t = \frac{\theta}{2}\left\{\sigma^2 \sum_{i=1}^{k-1} \frac{1}{(T-i+1)^2} + \sigma^2\left(1 - \frac{\log(T-k+1)}{T-k+1}\right)\right\}.$$

It turns out that in this case the presence of the new term adds on little insight as both factors multiplying σ^2, $\sum_{i=1}^{k-1} \frac{1}{(T-i+1)^2}$ and $1 - \frac{\log(T-k+1)}{T-k+1}$ are of the same order. Naturally, in the case of lack of correlation between the perception errors recasting the calculation in an explicit life-cycle setup proves to be futile as the overall consumption cost remains of the same order of magnitude. The situation is dramatically different if one allows for correlation between the error terms. Specifically, in the extreme case when the errors are perfectly correlated the cost takes the form

$$\rho_k^t = \frac{\theta}{2}\left\{\sigma^2 (\sum_{i=1}^{k-1} \frac{1}{T-i+1})^2 + \sigma^2\left(1 - \frac{\log(T-k+1)}{T-k+1}\right)\right\}.$$

Moreover, as k approaches T (the consumer approaches her .final periods of life) the expression can be approximated further with

$$\rho_k^t = \frac{\theta}{2}\left\{\sigma^2 \log^2 \frac{T}{2} + \sigma^2\right\}.$$

In this case the weights multiplying σ^2 are of different order. Specifically, the term that arises only in the life-cycle setup is of order $log^2 \frac{T}{2}$. The presence of this new higher order term is instrumental for the derivation of numerical results of this paper. Assuming for concreteness that the life span reaches T = 60 years and treating one year as one period it is straight forward to show that due to life cycle consideration the second order impact can increase up to by a factor of 13. While the overall cost of 0:008; (i.e., 0:8% of average consumption,) taking the original estimate of 0:0006 as given, remains still small in absolute value, especially if one acknowledges the assumption of extreme correlation between errors, it is clear that the presence of a life-cycle setup provides a new insight and can affect the numerical results by an order of magnitude.

The result is due to a very basic economic mechanism. At any point in time a given perception error not only inhibits consumption smoothing from one period to another it also influences the size of assets available in future periods. Consequently the change in the size of assets changes the entire future consumption path. Unquestionably the change is only marginal, as long as errors are not sizable, but this new variability of the consumption profile imposes additional second order costs of consumption volatility.

Moreover, if the perception errors are biased with mean _ then the expression describing the costs of fluctuations from the perspective of the k.th period of one's life can be approximated with

$$\rho_k^t = \mu \log \frac{T}{2} + \frac{\theta}{2} \left\{ (\sigma^2 + \mu^2) \log^2 \frac{T}{2} + (\sigma^2 + \mu^2) \right\}. \quad (8)$$

Clearly, as time progresses both the .first order term and the second order term increase. However, the speed of the expansion of the second order term exceeds the speed of the expansion of the first order term by the factor of $log \frac{T}{2}$. It appears that the new component of the second order term becomes asymptotically the dominant .figure and the costs related to the variability of the entire consumption profile constitute the principal burden of consumption volatility.

A Numerical Example

Assume that a consumer lives for T = 60 years and experiences N = 6 full cycles during her life time. Moreover, assume that the average duration of a recession is R = 2 years and that an expansion lasts on average for B = 8 years. Moreover, assume that the level of income is always .at and equal to y. However, whenever there is an expansion consumers expect their income to be higher in the future by ε percent and whenever there is a recession consumers expect their future income to be lower by - ε percent. In other words, it is assumed that the actual income is always .at and the business cycle affects only income expectations with recessions being times of pessimism and expansions being episodes of optimism. Finally, assume that cycles are fully deterministic with each recession followed by an expansion and so on.

The current framework requires agents to be indexed with two separate indices. One index corresponding to the age of a consumer and the other to the phase of the business cycle to which she was born. For illustrative purposes the paper traces the costs of .fluctuations to an individual born in the very first year of an expansion and to an individual born in the very .first year of a recession. Note that in the current setup the errors are NOT zero on average and the overall costs of .fluctuations do include a .first order term in addition to the expansion in the second order term.

Clearly, the pro.les are not smooth and the lack of smoothness exhibits two distinctive patterns. First of all, at a higher frequency the level of consumption displays significant upturns and downturns due to the existence of the business cycles (the changes of perception of future income streams.) In addition, at a higher frequency there exists a downward trend in the consumption profile. The presence of the trend is due to the fact that recessions are less frequent than expansions and on average agents spend too much over the business cycles, which affects their assets and consequently shift the permanent income consumption profile. Estimates of the actual costs of the lack of smoothness are given in the tables below.

Table 1. Numerical values of the costs

The value of ρ at Different Ages for an Agent Born in the First Year of an Expansion, $T = 60$, $\theta = 3$.

$\lvert\varepsilon\rvert$	1	20	30	40	50	55
1%	0.0001	0.0027	0.0043	0.0068	0.0102	0.0140
2%	0.0005	0.0056	0.0089	0.0134	0.0206	0.0283
5%	0.0037	0.0162	0.0243	0.0355	0.0533	0.0722
10%	0.0161	0.0410	0.0572	0.0793	0.1141	0.1509

The value of ρ at Different Ages for an Agent Born in the First Year of a Recession, $T = 60$, $\theta = 3$.

$\lvert\varepsilon\rvert$	1	20	30	40	50	55
1%	0.0001	0.0024	0.0042	0.0066	0.0108	0.0123
2%	0.0005	0.0057	0.0086	0.0135	0.0218	0.0247
5%	0.0037	0.0150	0.0237	0.0358	0.0564	0.0633
10%	0.0158	0.0385	0.0556	0.0796	0.1200	0.1330

Source: own computations

Several observation can be made based on the information contained in the tables above. First of all the actual costs do depend on the phase of the business cycle to which a given consumer was born. This is due the fact that errors made during recessions actually increase future wealth whereas errors committed during expansions negatively influence future assets. Moreover, the costs do increase with time. This is due to the fact that over time agents experience more downturns and upturns and consequently their consumption profile shifts away significantly from the original optimal path. This increase of the costs with time is due to the variation of the entire consumption profile in addition to the costs, analyzed in other contributions, due to period to period variability. As indicated in the table the actual cost rises by several orders of magnitude with consumer's age. However, the estimates provided are influenced by a first order effect, the .first term - falling in relative strength with time - in the context of equation 8, as the perception errors are not unbiased. The business cycle is assumed to be asymmetric with expansions appearing more often than recessions. Given that the direction of perception errors is positively correlated with the phase of the business cycle it is straightforward to observe that on average agents do overspend compared to their full information fi.rst best level of spending. Therefore, whenever, a recessions occurs agents cut their consumption dramatically for two reasons. First of all, during a recession agents are pessimistic the perception error is negative and agents consume less out of their future income. Secondly, given that on average they overspend their assets are lower than they otherwise would be and agents spend less out of their smaller assets. This basic feature is retained at the aggregate level as the perception errors are correlated across consumers and makes the aggregate consumption .fluctuate even though the level of income is always .at and each agent consumes on average the same quantity. Coincidently the variability of aggregate consumption relative to its mean matches the value reported by Lucas of 1:5% when the perception errors are equal in absolute value to 2%.

The paper now considers a minor extension of the above setup. As realistically consumers are impatient the need to provide relevant numerical estimates of the actual costs requires that the future consumption stream valuations be discounted. Accordingly, the paper modi.es the utility function to

$$U\left(\{c_t\}_{t=1}^{t=T}\right) = \sum_{t=1}^{T} \beta^{t-1} \frac{c_t^{1-\theta} - 1}{1-\theta}.$$

In addition, it is assumed that there exists a risk free return on assets equal to r and that in accordance with a literature cannon $\beta(1+r)=1$, which, in particular, assures in the absence of uncertainty the flatness of the consumption profiles. All remaining assumptions are unchanged. Under the modified assumptions consumer want to keep their consumption profiles flat with the level being determined by their current and their future expected income properly discounted. In particular, the level of consumption of a consumer born at time t in the i.th period of her life takes the form

$$c_{t+i-1} = y + x_{t+i-1} - \sum_{j=1}^{i} w_j x_{t+j-1},$$

where $w_j = \frac{1-\frac{1}{1+r}}{1-\left(\frac{1}{1+r}\right)^{T-j+1}}$ and x_{t+j-1} is the perception error of the future income stream from the perspective of period t + j - 1. The costs of consumption volatility from the perspective of the k'th period of one's life can be expressed as

$$(1-\rho_k^t)^{1-\theta} = \frac{1-\beta}{1-\beta^{T-k+1}} \sum_{i=k}^{T} \beta^{i-k} \left(1 + \varepsilon_{t+i-1} - \sum_{j=1}^{i} w_j \varepsilon_{t+j-1}\right)^{1-\theta},$$

where ε_{t+j-1}'s denote the percentage perception errors. Again limiting attention only to the first and second order terms the above expression can undergo additional simplifications to

$$\rho_k^t = \sum_{i=1}^{k-1} w_j \varepsilon_{t+j-1} + \frac{\theta}{2}\left((\sum_{i=1}^{k-1} w_j \varepsilon_{t+j-1})^2 + w_k \sum_{i=k}^{T} (1-w_i)\varepsilon_{t+i-1}^2 \right).$$

Finally, concentrating again only on the second order term, i.e., assuming temporarily unbiasedness of the errors and perfect correlation the cost term is given by

$$\rho_k^t = \frac{\theta}{2}(\sigma^2(\sum_{i=1}^{k-1} w_j)^2 + \sigma^2). \tag{9}$$

The last expression gives an approximation of the costs of consumption fluctuation when discounting is taken explicitly into account. At .first it may appear that an introduction of accounting should lower the costs estimates as future discrepancies do not influence value functions as distant future is discounted heavily with β^{t+i-1}. In fact quite the opposite is true. Indeed, future discrepancies are discounted and weigh less in the overall valuation, but at the same time future costs are discounted with identical weights and on net there is no direct impact arising from discounting. Moreover, the assumption of $\beta(1+r) = 1$ not only assures flat consumption profiles, but also implies that a nonzero interest is paid on assets. A given perception errors deviates the magnitude of assets from the optimal path and interest paid on the actual assets retained magnifies the discrepancy and increases the overall variability of assets and in term of the consumption profile. The new component of the second order term, $\sigma^2\left(\sum_{i=1}^{k-1} w_j\right)^2$ captures the overall effect of discounting. It is straight forward to establish that as soon as r > 0 the sum $\sum_{i=1}^{k-1} w_j$ exceeds $log\frac{T}{2}$. Moreover, in the special case when r = 0:05 per period the overall magnitude of the second order term is equal to 28 * 0:0006, i.e., 1:7% of the average consumption. While the absolute effect remains modest it is clear that the cost rises by an order of magnitude relative to the original Lucas's estimate. Tables below provide the actual costs, combining both the second and .first order terms, of consumption variability in the environment described at the beginning of this subsection with errors of perception being equal either ε or –ε depending on the phase of the business cycle.

Table 2. Numerical values of the costs – non zero interest rate

The value of ρ at Different Ages for an Agent Born in the First Year of an Expansion, $T = 60$, $\theta = 3, r = 0.05$.

$\|\varepsilon\|$	1	20	30	40	50	55
1%	0.0001	0.0069	0.0103	0.0142	0.0194	0.0239
2%	0.0005	0.0141	0.0209	0.0288	0.0390	0.0480
5%	0.0038	0.0375	0.0546	0.0742	0.0996	0.1288
10%	0.0171	0.0848	0.1193	0.1583	0.2084	0.2517

The value of ρ at Different Ages for an Agent Born in the First Year of a Recession, $T = 60$, $\theta = 3, r = 0.05$.

$\|\varepsilon\|$	1	20	30	40	50	55
1%	0.0001	0.0059	0.0094	0.0136	0.0192	0.0209
2%	0.0005	0.0122	0.0192	0.0274	0.0387	0.0420
5%	0.0037	0.0328	0.0503	0.0707	0.0987	0.1067
10%	0.0163	0.0748	0.1100	0.1506	0.2059	0.2210

Source: own computation.

The overall costs of consumption variability reach 10% of the average consumption at the age of 50 when the percentage perception error in absolute value is equal to 5% and attain 4% when the absolute value of the percentage perception error equals 2%. In addition, as in the previous case errors do increase with time. Unquestionably, the actual magnitudes while quantitatively orders of magnitude larger than the 0:06% estimate of Lucas are relatively small and are affected by the .first order term as the perception error are not unbiased. It appears fair to assert that most professional economists would consider biased errors of perception on average of 1:2% to be a remarkable proof of consumer rationality and consequently could consider the figures in the table above as reliable. Nevertheless, there is a need to obtain sounder .figures that would correspond to fully rational behavior of consumers. The following section recasts the problem in an explicit rational framework, i.e., it corrects the above estimates and purges from the first order term.

Rational Framework

The objective of this section is to provide quantitative estimates of output .fluctuations in a life cycle framework with stochastic income and fully rational behavior. The costs of fluctuations arise as consumption profiles are smooth only in expected terms and ex post agents always either undersave or oversave and consequently their assets vary and shift entire consumption pro.les. The basic setup is the same. The paper now abandons discounting and returns to the assumption of zero net real interest rate. In addition, the felicity function is modified to facilitate analytical tractability. Specifically, it is assumed that agents live for T periods and their utility takes the form, $\bar{c}$ is an unspecified constant at this stage,

$$U\left(\{c_t\}_{t=1}^{t=T}\right) = \sum_{t=1}^{T} \frac{(c_t - \bar{c})}{\bar{c}^{\theta}} - \frac{\theta}{2}\frac{(c_t - \bar{c})^2}{\bar{c}^{1+\theta}}.$$

Throughout her life time an agent receives a vector of stochastic income streams $\{y_t\}_{t=1}^{T}$ and starts her life with no assets. Under these assumptions the level of consumption remains .at albeit in expected terms $c_t = E_t c_{t+i}$ and the level of consumption pinned down by the budget constraint is given by

$$c_t = \frac{1}{T}\sum_{i=1}^{T} E_0 y_i + \sum_{i=1}^{t} v_i \sum_{j=i}^{T} \varepsilon_i^j$$

where $v_i = \frac{1}{T-i+1}, \varepsilon_i^j = E_i y_j - E_{i-1} y_j,$ and E_0 denotes the expectation operator from the perspective of the period before the period of the agent's birth. In a stochastic environment the value of realized utility depends on specific realizations of income. Nevertheless, it is possible to assess the expected cost of fluctuations as of the k'th period of a consumer life form the ex ante perspective as well unconditional perspective.

In assessing the costs the paper .first turns to a number of specific and yet suggestive examples. First let's consider the case when the income process takes the canonical time series form

$$y_t = \bar{y} + u_t,$$

with ut of mean zero, variance σ^2 and serially uncorrelated. Moreover, let the constant $\bar{c}$ in the utility function be equal the average value of income $\bar{y}$. Note that the unconditional variance of income relative to its mean is given by $\sigma^2_{\bar{y}^2} = \frac{\sigma^2}{\bar{y}^2}$. When the income process takes this specific form the expression describing the level of consumption in the period k of a consumer, born at time t life time is given by

$$c^k_{t+k-1} = \bar{y} + \frac{u_t}{T} + \frac{u_{t+1}}{T-1} + \ldots + \frac{u_{t+k-1}}{T-k+1}.$$

Naturally, the level of consumption is not fixed and depends on specific realizations of income shocks. Nevertheless, it is possible to assess the expected level of realized utility. Let E denote the unconditional expectation operator. The level of expected utility realized in the .final period of the consumer lifetime is given by

$$U^T_R = E\frac{(c_t - \bar{y})}{\bar{y}^\theta} - \frac{\theta}{2}E\frac{(c_t - \bar{y})^2}{\bar{y}^{1+\theta}},$$

which can be simplified to

$$U^T_R = -\frac{\theta}{2}\frac{\left(\sum_{i=1}^T \frac{1}{i^2}\right)}{\bar{y}^2}\sigma^2.$$

As the consumption profile is not flat specific income realizations impose a burden on the consumer. The expected, in the unconditional sense, costs of fluctuations to a T periods old consumer can be found, limiting one's attention only to first order expansion in ρ^t_T in the standard manner and are given by

$$\rho^t_T = \frac{\theta}{2}\left(\sum_{i=1}^T \frac{1}{i^2}\right)\frac{\sigma^2}{\bar{y}^2}.$$

Two qualifications must be made immediately. First of all the above expression gives the percentage loss in consumption to a T periods old consumer as compared to the case with no income fluctuations at all, i.e., income equal to its unconditional mean $\bar{y}$ at all periods. Secondly, in this example life cycle accounting increases the costs as the costs do increase with T. However, the increase is very mild as the series $\sum_{i=1}^T \frac{1}{i^2}$ converges to a modest magnitude of $\frac{\pi^2}{2}$. Nevertheless, to assess the costs fully one must put them in the proper metric. In particular, one must assign a value to $\frac{\sigma^2}{\bar{y}^2}$. In general, the assignment is not trivial as there is no basis to assume that individual consumption, aggregate consumption, and aggregate income variability are identical. In particular, in the present context the aggregate level of consumption at time t can be expressed as

$$C_t = T\bar{y} + u_t \sum_{i=1}^{T} v_i + u_{t-1} \sum_{i=1}^{T-1} v_i + \ldots + u_{t-T+1} \sum_{i=1}^{1} v_i.$$

Consequently, the unconditional variance of the aggregate consumption relative to its mean is given by

$$\sigma_L^2 = \frac{2T - \left(\sum_{i=1}^{T} \frac{1}{i}\right)}{T^2} \frac{\sigma^2}{\bar{y}^2}.$$

Treating the expression σ_L^2 as observable and equal to $(0.015)^2$ it is possible to put a final assessment of the expected costs of fluctuations to a T periods old consumer

$$\rho_t^T = \frac{\theta}{2}\sigma_L^2 \frac{T^2\left(\sum_{i=1}^{T} \frac{1}{i^2}\right)}{2T - \left(\sum_{i=1}^{T} \frac{1}{i}\right)}.$$

Specifically, when T is equal to 60 the costs increase by a factor of 50 and are equivalent in total to 3% of consumption. Naturally, these are only the expected costs and the actual costs do depend on specific realizations. Nevertheless, increase in expected terms is dramatic. It must be acknowledged that the sharp increase in the costs relative to the Lucas's estimate comes in the present context to a large extent from a sizeable discrepancy between the aggregate and individual consumption volatility and directly not so much from the explicit lifecycle modeling. Moreover, the variance of aggregate output relative to the variance of aggregate consumption is given by

$$\frac{\sigma_C^2}{\sigma_Y^2} = \frac{\left(2T - \left(\sum_{i=1}^{T} \frac{1}{i}\right)\right)\sigma^2}{\sigma^2}$$

and exceeds, for the value of T equal to 60, the observed empirical value of 0.5. The following examples explore the issues further.

The basic setup remains unchanged. Again, the constant $\bar{c}$ in the utility function is set to be equal to the unconditional mean of income $\bar{y}$. However, the process generating income is modified and takes the form of a Markov chain with two possible realizations of income y_H and y_L and the transition matrix given by

Table 3. Transition probabilities

	H	L
H	□HH	□HL
L	□LH	□LL

Source: own assumptions

Naturally, $a_{H*} = 1\ and\ a_{L*} = 1$ and all entries are between zero and one. Under these assumptions the unconditional likelihood of state H appearing is given by $p_H = \frac{a_{HL}}{a_{HL}+a_{LH}}$ and the likelihood of state L appearing is given by $p_L = \frac{a_{LH}}{a_{HL}+a_{LH}}$. The unconditional mean and variance of income can be respectively expressed as $\bar{y} = p_H y_H + p_L y_L$ and $\sigma_{\bar{y}}^2 = p_H p_L (y_H - y_L)^2$. Again, for notational simplicity the paper focuses on cost to a T periods old consumer. Observe that the level of consumption of a T periods old consumer born at time t at time t + T - 1 takes the form

$$c_{t+T-1}^{T} = \frac{\sum_{i=1}^{T} E_{t-1} y_{t+i-1}}{T} + \sum_{i=1}^{T} v_i \sum_{j=i}^{T} \varepsilon_{t+i-1}^{t+j-1},$$

where again $v_i = \frac{1}{T+i-1}\ and\ \varepsilon_{t+i-1}^{t+j-1} = E_{t+i-1} y_{t+j-1} - E_{t+i-2} y_{t+j-1}$. Observe that the unconditional mean of c_{t+T-1}^{T} is not surprisingly equal to $\bar{y}$. Therefore, the expected costs of fluctuations to a T periods old consumer can be expressed as

$$\rho_t^T = \frac{\theta}{2} \frac{E\left(c_{t+T-1}^{T} - \bar{y}\right)^2}{\bar{y}^2},$$

i.e., the entire problem amounts to a calculation of the unconditional variance of c_{t+T-1}^{T}.

Letting k approach T, the end of a life time, the expression can be simplified further to

$$\rho_T = \frac{\theta}{2}\left(\left(\frac{1+\lambda}{1-\lambda} \sum_{i=1}^{T-1} \left(\frac{1-\lambda^{T-i+1}}{T-i+1}\right)^2\right) \sigma_{\bar{y}^2}^2 + \sigma_{\bar{y}^2}^2 \right). \qquad (10)$$

It is clear that the expression 10 contains two distinctive terms. One, $\sigma_{\bar{y}^2}^2$ corresponding to the original Lucas's approach and the other new and due to life cycle accounting. The new term is a function of both T and λ. For any fixed λ the new term is bounded from above as the series $\sum_{n=1} \frac{1}{n^2}$ is convergent. Moreover, for a given T the new term is a non-monotone function of λ and reaches zero when λ=1 and $\sum_{n=1}^{T-1} \frac{1}{n^2}$ when λ=0. Obviously, as λ approaches zero the expression 10 remains of order$\sigma^{2}_{y^{-z}}$ and life cycle accounting brings no new insight. On the other hand if λ differs from zero the new term can reach sizeable magnitudes and life cycle accounting could increase the overall costs by up to the factor of 30. Note, however, that given the above process the unconditional variance of income tends to infinity as λ approaches 1. Nevertheless, it is clear that explicit life cycle accounting captures allows to account for additional variability of the entire consumption pro.les. It should be emphasized that the realized costs could be much higher than the expected when given realizations are particularly unfavorable.

Moreover, if the actual income generating process is more complex the situation can be even more dramatic. Specifically, if one designs a process with a stochastic trend and a transitory component then the actual impact of uncertainty on assets and consequently on the costs can be much higher. At any point in time agent do not know how to interpret a given innovation of income they must use Bayes rule to update and consequently over react to transitory innovations and underreact to permanent innovations. This rises the overall uncertainty of imposes additional costs.

Conclusions

The paper explores a very basic mechanism and shows that consumption fluctuations can be costly to an individual. Specifically, in an explicit life cycle setup it is shown that the costs of consumption volatility increase in expected terms with age and can be as high as 10% of the average consumption to a fifty year old consumer. The result is novel for two reasons. Both the quantitative estimate and the underlying economic mechanism that drives the results are new to the literature despite the overall technical simplicity of the paper. In particular, unlike other contributions in the field the paper does not invoke concepts of heterogeneity, market incompleteness, market imperfections or leisure-consumption choice and tries to fill the technical gap between the simplistic framework of Lucas [6] and the modern era tool box approaches. Specifically, the paper focuses on redoing the original calculation of Lucas in an environment with fixed time horizon rather than in an infinite horizon setup and consequently it uses individual consumption volatility rather than the aggregate figures to gauge the actual costs of output fluctuations to an individual rather than to a representative infinitely lived consumer. The change of the environment appears to have a profound effect on the quantitative estimates rising them by orders of magnitude.

The underlying economic mechanism explored in the paper is quite simple with the volatility of the entire consumption pro.les being at its root. Rational economic agents equalize the marginal utility of the current consumption with the expected future marginal utility of consumption. However, this specific form of behavior is efficient only in ex ante sense. Ex post, as soon as uncertainties are resolved, it is either the case that the amount saved in the previous period is too high or too low depending on a specific realization of the current income. This inefficiency ex post makes economic agents revise their entire consumption plans in accordance with their current expectations of future income and available assets. Therefore, uncertainty impacts a given consumer via two different channels. First of all, it prevents consumption smoothing in realized terms, the notion extensively explored in other contributions, and consequently impacts consumer welfare. Secondly, it introduces variability of the entire consumption plans, the key notion explored in this paper. Therefore, to an individual a given realization of the current income implies that not only the current level of consumption is different from the expected one, but also that the same is true for all future periods, the new expected level of consumption is different from the ones previously expected. In other words, the randomness of the current income in addition to its direct impact on consumption it introduces uncertainty to future assets. Naturally, on average the change in future assets is zero in a rational framework, however, the second order, resulting from the variance term, effect is present and impacts negatively consumer welfare. Moreover, in general the magnitude of the variance term expands with time as there are more and more periods in which the realized income was different from the expected one. Consequently, in expected terms the expected, in ex ante sense, welfare costs of .fluctuations to fifty year old consumer can exceed the costs to a twenty year old consumer.

Numerically, the estimates of the costs of fluctuations differ from the original estimate of Lucas of 0.06% of consumption by orders of magnitude. Moreover, the actual cost do increase with time in proportion of $log^2 \frac{T}{2}$ with potential of reaching 10% of consumption at the age of fifty. The quantitative estimates are by no means considered as final. On the contrary, the paper is thought of as the one replacing Lucas's [6] contribution as the point of departure and provides only a benchmark estimate and standard extensions that have followed since the seminal paper of Lucas [6] including accounting for indirect channels, Gali et. al. [4], are very likely to lift the cost upward further.

REFERENCES:

1. Alvarez F, and U. Jermann (2000), Using Asset Prices to Measure the Cost of Business Cycle,.NBER Working Paper # 7978.
2. Atkeson, A. and C. Phelan (1994), Reconsidering the Costs of Business Cycles with Incomplete Markets..NBER Macroeconomic Annual 9: 187-207.
3. Ball, L. and D. Romer (1991), Real Rigidities and the Non-Neutrality of Money, Review of Economic Studies, 57, 183-203, Reprinted in Mankiw and Romer (1991).
4. Gali, J., M. Gertler and J. Lopez-Salido (2002), .Markups, Gaps and the Welfare Costs of Business Fluctuations,.NBER Working Paper #8850.
5. Imrohoroglu, A. (1989), Costs of Business Cycles with Indivisibilities and Liquidity Constraints, Journal of Political Economy, 97, 1664-1683.
6. Lucas, R. E., Jr. (1987), Models of Business Cycles, Basil Blackwell, Oxford.
7. Lucas, R. E., Jr. (2003), Macroeconomic Priorities,. American Economic Review, 93, 1-14.
8. Meltzer, A. (1988), Keynes's Monetary Theory: A Different Interpretation, Cambridge University Press, Cambridge.
9. Pallage S. and M. Robe (2000), Magnitude X on the Richter Scale: Welfare Costs of Business Cycles in Developing Countries, CREFE Working Paper #124.
10. Storesletten, K., C. Telmer, and A. Yaron (2001), The Welfare Costs of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk, European Economic Review, 45, 1311-1339.
11. Vandenbroucke G. (1998), The Welfare Costs of Fluctuations. How Far Should We Go? Paper Presented at CEPRMAP Stochastic Simulation Workshop.