DISTANCE LEARNING AND STUDY SUPPORTS OF DISTANCE EDUCATION

Practical demonstration of a seminar called 'Economic Evaluation of Public Projects of Investment Character' in the subject of 'Public Economics'

INTRODUCTION

Distance learning within the framework of a composite form of bachelor and master degree study programs has become a trend at Czech universities recently. Distance learning is based, above all, on the students' individual work with special teaching texts, which are being developed for the purposes of distance education.

The personal contact between the student and the tutor is very limited. There are only three one-hour consultations (initial, running, and final) during the course of a term. The communication between the tutor and the student is based, above all, on communication by e-mail. And that is why the study support of distance education is, from both the content and formal points of view, of a specific form. For this purpose, they are created in the way to show the information in a clear and concentrated form. Moreover, the tutor acts within the framework of the entire study material as the student's guide in the form of advice, remarks and comments.

And how are these study texts drawn up? Each chapter starts with setting up an objective that the students should meet - it says clearly what the students should know after they have studied the given chapter. A study guide that explains how the students should work follows. Then, the attention is paid to the curriculum itself. It is rich in the quantity of practical examples enabling better understanding and remembering of acquired information. Each chapter is closed with a summary and a list of terms to learn. There are also revisionary questions to check how the students have mastered the given problems. Other checking components of the texts are tests with a key. The very end of each chapter brings a list of used literature, which can be used if the student would like to study the given problems in more detail.

The authors of this kind of study books pay special attention to the graphic aspect. The study texts are, for example, supplemented with pictographs and marginal comments. For pictographs used by Faculty of Economics and Administration, the University of Pardubice, see the table.
The study support texts for distance education are drawn up both as textbooks and as workbooks. The study support - workbooks usually contain just some of the above distance components.

The following part presents a concrete demonstration of a seminar in the subject of 'Public Economics', the topic of which being 'Economic Evaluation of Public Projects of Investment Character'. The subject of 'Public Economics' is taught at Faculty of Economics and Administration, the University of Pardubice, in the second year of a composite form of bachelor study programs called 'Economy Politics and Administration' and 'System Engineering and Information Science'.

**ECONOMIC EVALUATION OF PUBLIC PROJECTS OF INVESTMENT CHARACTER**

**Objectives:**
After studying this chapter, you should have a good understanding of:

⇒ how to determine the present value of multiple (irregular) and annuity cash flow;
⇒ how to evaluate investment public project using net present value of benefits, benefit-cost ratio and internal rate of return;
⇒ how to explain relationships among net present value of benefits, benefit-cost ratio and internal rate of return.

**Guide of Study:**
Now you will solve a set of examples that will help you to acquire the methods of evaluation of public projects with investment character. Attention is paid to the most important criteria of evaluation of investment public projects - net present value of benefits, benefit-cost ratio and internal rate of return. The relationships among these criteria are also explained.

*Good luck during your study!*

**PRESENT VALUE AND DISCOUNTING**
In case that the costs and benefits of projects are created during a longer time period (longer than one year) - in case of investment projects, it is necessary to respect time value of money.
In the most general sense, the phrase time value of money refers to the fact that an EUR in hand today is worth more than an EUR promised at some time in future. To guarantee comparability of all costs and benefits, it's necessary to calculate the present value of a future cash flow of costs and benefits and determine their today value.

**Concept Question**
Do you remember what you learned about the calculation of the present value in „Financial Mathematics“? How is it possible to calculate the present value of a cash flow?

Present value = future value * present value factor

\[ \text{present value factor (discount factor)} = \frac{1}{(1 + i)^n}, \]

where

- i ... discount rate (interest rate),
- n ... number of years (number of periods).

Present value of multiple irregular cash flow =

\[ \sum_{t=1}^{n} \frac{CF_t}{(1 + i)^t}, \]

where

- CF_t ... cash flow in single years,
- i ...... discount (interest) rate,
- t ...... single years,
- n ...... total number of years.

Present value of annuity cash flow - we can calculate present value of annuity cash flow not only with present value factor but also with annuity present value factor.

Present value of annuity cash flow = annuity * annuity present value factor

\[ \frac{1 - \frac{1}{(1 + i)^n}}{i}, \]

where

- i ... discount (interest) rate,
- n ... total number of years.

**EXAMPLE 1**

**Problem**
Calculate present value of the following annuity cash flow, where discount rate is 7 %. Calculate using present value factor and annuity present value factor and then compare the results.

<table>
<thead>
<tr>
<th>Period</th>
<th>Cash flow in EUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st year</td>
<td>1,200.00</td>
</tr>
<tr>
<td>2nd year</td>
<td>1,200.00</td>
</tr>
<tr>
<td>3rd year</td>
<td>1,200.00</td>
</tr>
</tbody>
</table>
Complete the following claims:

a) Present value of the cash flow calculated using present value factor is equal to ...................................... EUR.

b) Present value of the cash flow calculated using annuity present value factor is equal to ...................................... EUR.

c) Present value of the cash flow calculated using present value factor is ........................................ (higher than, lower than, equal to) present value of the cash flow calculated using annuity present value factor.

**Solution**

**a) Present value of the cash flow calculated using present value factor**

<table>
<thead>
<tr>
<th>Period</th>
<th>Cash flow in EUR</th>
<th>i = 7%</th>
<th>PV of cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st year</td>
<td>1,200,000</td>
<td>1,200,000 * 0.11222</td>
<td>1,122,000</td>
</tr>
<tr>
<td>2nd year</td>
<td>1,200,000</td>
<td>1,200,000 * 0.10476</td>
<td>1,047,600</td>
</tr>
<tr>
<td>3rd year</td>
<td>1,200,000</td>
<td>1,200,000 * 0.09792</td>
<td>979,200</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>3,148,800</td>
</tr>
</tbody>
</table>

Present value of the cash flow calculated using present value factor is equal to 3,148,800 EUR.

**b) Present value of the cash flow calculated using annuity present value factor**

\[
PVA = \text{annuity} \times \text{APVF} = \text{annuity} \times \frac{1 - \left(1 + \frac{1}{i}\right)^{-n}}{i}
\]

\[
PVA = 1,200,000 \times \text{APVF} (7\% , 3\text{ years}) = 1,200,000 \times 2.624 = 3,148,800
\]

Present value of the cash flow calculated using annuity present value factor is equal to 3,148,800 EUR.

c) Present value of the cash flow calculated using present value factor is equal to present value of the cash flow calculated using annuity present value factor.

**RULES OF EVALUATION OF INVESTMENT PROJECTS**

In case of evaluation of investment public projects when it is necessary to respect time value of money, we can use the following criteria:

- net present value of benefits;
- benefit-cost ratio;
- internal rate of return.

**Net Present Value of Benefits and Benefit-Cost Ratio**

Net present value of benefits (NPV) represents the difference between the present value of benefits and costs. The project is acceptable if net present value of benefits is \( \geq 0 \). The project with the highest net present value of benefits is the most favorable.
Net present value of benefits = \[ \sum_{t=1}^{n} (B_t - C_t) \times (1 + i)^t \] 

\[ \sum_{t=1}^{n} \], where

- \( B_t \) ... benefits of the project in single years,
- \( C_t \) ... costs of the project in single years,
- \( i \) ... discount rate,
- \( t \) ... single years,
- \( n \) ... total number of years.

**Benefit-cost ratio (BCR)** - the project is acceptable if benefit cost ratio is \( \geq 1 \). The project with the highest benefit cost ratio is the most favorable.

\[
\frac{\sum_{t=1}^{n} B_t (1 + i)^t}{\sum_{t=1}^{n} C_t (1 + i)^t}
\]

\[ \sum_{t=1}^{n} \], where

- \( B_t \) ... benefits of the project in single years,
- \( C_t \) ... costs of the project in single years,
- \( i \) ... discount rate,
- \( t \) ... single years,
- \( n \) ... total number of years.

**EXAMPLE 2**

**Problem**

Compare the following projects.

<table>
<thead>
<tr>
<th></th>
<th>Capital costs (EUR)</th>
<th></th>
<th>Capital costs (EUR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>150,000</td>
<td>D</td>
<td>150,000</td>
</tr>
<tr>
<td></td>
<td>Benefits (EUR)</td>
<td>Operating costs (EUR)</td>
<td>Benefits (EUR)</td>
</tr>
<tr>
<td>1st year</td>
<td>20,00</td>
<td>100,00</td>
<td>1st year</td>
</tr>
<tr>
<td>2nd year</td>
<td>20,00</td>
<td>100,00</td>
<td>2nd year</td>
</tr>
<tr>
<td>3rd year</td>
<td>300,00</td>
<td>100,00</td>
<td>3rd year</td>
</tr>
<tr>
<td>4th year</td>
<td>300,00</td>
<td>100,00</td>
<td>4th year</td>
</tr>
</tbody>
</table>

Set:

a) Net present value of benefits of project C, if the discount rate is 8 %.
b) Benefit cost ratio of project C, if the discount rate is 8 %.
c) Whether project C is acceptable, if the discount rate is 8 %.
d) Net present value of benefits of project D if the discount rate is 8 %.
e) Benefit cost ratio of project D, if the discount rate is 8 %.
f) Whether project D is acceptable, if the discount rate is 8 %.
g) Which of these projects is more advantageous?
## Solution

### Project C

<table>
<thead>
<tr>
<th>Cash flow</th>
<th>PV of cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital costs</td>
<td>150,000</td>
</tr>
<tr>
<td>Operating costs - 1st year</td>
<td>100,00</td>
</tr>
<tr>
<td>Operating costs - 2nd year</td>
<td>100,00</td>
</tr>
<tr>
<td>Operating costs - 3rd year</td>
<td>100,00</td>
</tr>
<tr>
<td>Operating costs - 4th year</td>
<td>100,00</td>
</tr>
<tr>
<td>PV of total operating costs</td>
<td>331,200</td>
</tr>
<tr>
<td>Benefits - 1st year</td>
<td>18,520</td>
</tr>
<tr>
<td>Benefits - 2nd year</td>
<td>17,140</td>
</tr>
<tr>
<td>Benefits - 3rd year</td>
<td>238,200</td>
</tr>
<tr>
<td>Benefits - 4th year</td>
<td>220,500</td>
</tr>
<tr>
<td>PV of benefits</td>
<td>494,360</td>
</tr>
<tr>
<td>Net present value</td>
<td>1,027</td>
</tr>
</tbody>
</table>

#### Benefit-cost ratio

| Benefits-cost ratio | 13,160 |

a) Net present value of benefits of project C is equal to 13,160 EUR.
b) Benefit cost ratio of project C is equal to 1.027.
c) YES, project C is acceptable.
d) Net present value of benefits of project D is equal to 2,900 EUR.
e) Benefit cost ratio of project D is equal to 1.010.
f) YES, project D is acceptable.
g) Project C is more advantageous.

### Project D

<table>
<thead>
<tr>
<th>Cash flow</th>
<th>PV of cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital costs</td>
<td>150,000</td>
</tr>
<tr>
<td>Operating costs - 1st year</td>
<td>0</td>
</tr>
<tr>
<td>Operating costs - 2nd year</td>
<td>0</td>
</tr>
<tr>
<td>Operating costs - 3rd year</td>
<td>79,400</td>
</tr>
<tr>
<td>Operating costs - 4th year</td>
<td>73,500</td>
</tr>
<tr>
<td>PV of total operating costs</td>
<td>152,900</td>
</tr>
<tr>
<td>Benefits - 1st year</td>
<td>0</td>
</tr>
<tr>
<td>Benefits - 2nd year</td>
<td>0</td>
</tr>
<tr>
<td>Benefits - 3rd year</td>
<td>158,800</td>
</tr>
<tr>
<td>Benefits - 4th year</td>
<td>147,000</td>
</tr>
<tr>
<td>PV of benefits</td>
<td>305,800</td>
</tr>
<tr>
<td>Net present value</td>
<td>1,010</td>
</tr>
</tbody>
</table>

#### Benefit-cost ratio

| Benefits-cost ratio | 2,900 |

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Internal Rate of Return

**Internal rate of return** (IRR) calls for determining the yield on an investment, that is, calculating the discount rate that equates the cash outflows (costs) of an investment with the subsequent cash inflows (benefits). The final selection of any project under the internal rate of return method will depend on the yield exceeding some minimum level, such as required discount (interest) rate. The project is acceptable if the internal rate of return is higher than the required interest rate and then the most advantageous project is the project with the highest internal rate of return.

\[
\sum_{t=1}^{n} (B_t - C_t) * (1 + r)^{-t} = 0
\]

, where

- \( B_t \) ... benefits of the project in single years,
- \( C_t \) ... costs of the project in single years,
- \( r \) ... internal rate of return,
- \( t \) ... single years,
- \( n \) ... total number of years.

To find an internal rate of return we can use **interpolation**. Since the internal rate of return is determined when the present value of the inflows (benefits) equals the present value of the outflows (costs), we need to find a discount rate that equates the present value of the benefits with the present value of the costs.

We can use the following formula:

\[
\text{IRR} = i_L + \frac{\text{NPV}_L}{\text{NPV}_L + |\text{NPV}_H|} (i_H - i_L)
\]

, where

- IRR ..... internal rate of return,
- \( \text{NPV}_L \) ... net present value of benefits for lower interest rate,
- \( \text{NPV}_H \) ... net present value of benefits for higher interest rate,
- \( i_L \) ... lower selected interest rate (which will guarantee positive NPV),
- \( i_H \) ... higher selected interest rate (which will guarantee negative NPV).

**EXAMPLE 3**

**Problem**
Calculate internal rate of return of project D (see example 2) and decide whether the project is acceptable. Required interest rate is 6 %.

**Solution**

\[
\text{IRR} = i_L + \frac{\text{NPV}_L}{\text{NPV}_L + |\text{NPV}_H|} (i_H - i_L)
\]
If $i = 8\%$, then $NPV = 2,900$ EUR  $\Rightarrow i_L = 8\%$ and $NPV_L = 2,900$ EUR.

If $i = 9\%$, then $NPV = -2,000$ EUR $\Rightarrow i_H = 9\%$ and $NPV_H = -2,000$ EUR.

$$IRR = 8 + \frac{2,900}{2,900 + |-2,000|} (9 - 8)$$

$$IRR = 8.6\%$$

Internal rate of return is equal to 8.6%. The project is acceptable if the required interest rate is equal to 6% (8.6% > 6%).

**Relationships among Net Present Value of Benefits, Benefit-Cost Ratio and Internal Rate of Return**

During the process of evaluation of investment public project it is necessary to take into consideration the following facts:

- if the net present value of benefits is positive ($> 0$), then the benefit-cost ratio is $> 1$;
- if the net present value of benefits is equal to 0, then the benefit-cost ratio is equal to 1;
- if the net present value of benefits is negative ($< 0$), then the benefit-cost ratio is $< 1$;
- internal rate of return is a rate where net present value of benefits is equal to zero.

**EXAMPLE 4**

**Problem**

Select the most advantageous public project from the following projects:

a) project A - if the interest rate is 8%, net present value of benefits of this project is equal to minus 125,000 EUR;

b) project B - the internal rate of this project is equal to 8.5%;

c) project C - if the interest rate is 8.5%, benefit-cost ratio of this project is equal to 1.1;

d) project D - if the interest rate is 8%, net present value of benefits of this project is equal to 0.

All these projects are connected with capital cost in amount of 500,000 EUR.

**Solution**

The most advantageous project is **project C** because:

- project A - if the interest rate is 8%, $NPV = -125,000$ EUR, then the project is not acceptable;
- project B - $IRR = 8.5\%$, so that if the interest rate is 8.5%, then $NPV = 0$ and $BCR = 1$;
- project C - if the interest rate is 8.5%, $BCR = 1.1$, then at the interest rate 8.5%, $NPV > 0$;
- project D - if the interest rate is 8%, $NPV = 0$ (project D is worse than both project C and B).
LIST OF TERMS

Discounting
Discount rate
Net present value of benefits
Benefit-cost ratio
Internal rate of return

LITERATURE


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