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THE APPLICATION OF GAME THEORY INTO THE MODEL OF SIGNALLING

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Introduction

In the article we analyze signaling i.e. the situation where informed entities send signals to uninformed ones. A signal can convey information which creates new decisions and new strategies. We examine decision making process on the base of available signals in a game under uncertainty.

Thus, the main aim of this article is to show, how to model a problem of transforming a signal into information in a way that shows the causes and results of it. For that reason we use the theory of games which enables to acquire a better understanding of the decision making process in such a situation. In this article we use a version of the beer-quiche game of Kreps.

The first stage of research task realization is to determine the assumptions for the model of signaling. Then we show the way to introduce a problem of signaling using of game theory's instruments. The next stage of research is to present the signal transformation into information. The last stage is to show and discuss the possible developments of this model.

Access to the information in economics

In 60. and 70. of XX century economists were analyzing the impact of the access to information on a decision process. They indicated information as the variable which is essential for the establishing the equilibrium. Among the economists it was J. A. Mirrlees who introduced the concept of asymmetric information¹. The theory was developed by G.A. Akerlof (Akerlof, 1970), M. Spence (Spence, 1973) and J. E. Stiglitz (Stiglitz, 1975; Rothschild, Stiglitz 1976). Their publications were the breakthrough in economics².

The economics of information has evolved over the past century. Nowadays the importance of information in economics is widely known. Entities, which possess informational advantage towards the others, can use it to achieve their particular aims. According to the available information, firms can make higher or lower profits. The most important problems in the field of information now are the availability and the credibility. The reason is that some of the entities can achieve comparative advantage towards the others according to the additional information. Access to the additional, private information enables them not only to improve their decisions, but also to manipulate information.

As a result unequal access to information for all entities became the crucial idea in the economics of information. This concept is called the asymmetric information. There is a wide range of problems connected with asymmetric information, for example moral hazard, signaling, adverse selection and screening.

The problems with access to information generate conditions of imperfect, incomplete, uncertain or asymmetric information. So, these are violating the conditions of perfect, complete, certain or symmetric information. Perfect information is a term used to describe a state of complete knowledge about the actions of other players. Moreover, this knowledge should be instantaneously updated as new information arises. The complete information game, according to the old definition, it is a game where each player knows exactly the rules. In other words each player starting the game knows the payoffs and strategies available to other players, while not necessarily has knowledge about the action of the other players inside the game. Prisoner Dilema is the example of complete but imperfect information. The player knows the payoffs and strategies available to other players, however does not know the action of the other player. There are two kinds of games with complete but imperfect information. The first

¹ He published the economic models and equations which centered around the situations of asymmetrical or incomplete information. Mirrlees and William Vickrey collaborated on the principles of *moral hazard* and *optimal income taxation*. In 1996 they shared the Nobel Prize for Economics "for their fundamental contributions to the economic theory of incentives under asymmetric information".

² They shared the 2001 Nobel Prize for Economics.

are games with simultaneous moves, and the second are games where, late in the game, nature makes moves and those moves are not immediately revealed to all players. Moreover, any game of incomplete information can be transformed into a game of imperfect information by including nature as a player in the game and conditional payoffs for nature's unknown moves³. According to the new definition of complete games from Rasmusen, in such games, nature is not allowed to make the first movement, although other players can observe this movement⁴ (Rasmusen, 2006).

The concepts of incomplete information and of asymmetric information are not equivalent. However many games of incomplete information games are also games with the informational advantage of one player i.e. with asymmetric information. If there is no initial move by the nature, but the first player takes a move unobserved the second player, and the first player moves again later in the game, it is the game of asymmetric but complete information. It is connected with the principal-agent problem, when the agent possesses additional or private information, which is unavailable to the principal, not even at the end of the game. Incomplete or asymmetric information in the game theory concerns the lack of access to information about the rival's strategies, payoffs or the payoffs' functions (Rasmusen, 2006).

Uncertain information is used to describe a game in which players do not know exactly what kind of game they are playing. They know what payoff of playing a particular strategy will be, given the strategies played by others. The game of incomplete (or asymmetric information) can be transformed into the game of uncertain information by applying the assumption, that nature chooses the sequence of player's movements. On the other hand the game of uncertain information can be transformed into the game of incomplete (or asymmetric information) by randomizing. Randomizing is ascribing expected probabilities to the different types of the rival or to his choices (strategies). The player has to determine probabilities according to available information, his previous experience, presumptions or intuition (Carmichael, 2005). As a result he eliminates nature from a game, so the game becomes a certain one.

Game theory has developed in parallel with the theory of asymmetric information. As a result it influences asymmetric information research. Describing the problem of the access to the information in words of the game theory, symmetric information is a situation, in which none of the players has an informational advantage over the others. According to E. Rasmu-

³ The games of incomplete information, known as *I-games*, were the fields of research for J. Harsányi, who awarded a Nobel Price.

⁴ Simultaneous games and the games with the move of nature at some stage of the game, which is not observed

sen, in the game of symmetric information the informational set of a particular player in any node, where the player makes decision or in the end node contains at least the same elements that informational sets of other players (Rasmusen, 2006).

The information set for a particular player establishes all the possible moves that could have taken place in the game so far. In words of game theory the information set for player is a set of nodes available to one player in the given stage of the game, but on different paths. The player's information set includes only nodes at which this player moves. One node cannot belong to two different information sets of the player. Information sets also show the effects of unobserved moves by nature. If a game has complete information, each single node constitutes an information set. In fact it is the point that a player actually reached at the given stage of the game. (Malawski, Wiczorek, Sosnowska, 2004). In a game of incomplete information the information set consists of several nodes. However player does not know the exact location the game has reached in the game tree. In such a game it is often nature that moves the first.

Games with the moves of nature can also provide symmetrical access to information. If any player has additional, private information, he can use this asymmetric information to gain a comparative advantage. Moreover, each player can use signalling to provide other players the concrete information called signal. The signal is an intentional message containing a portion of information, which purpose is to make other players (receivers of the signal) to act in a certain way (Macho-Stadler, Pérez-Castrillo, 1997). In other words a signal can convey information which creates new decisions and new strategies. A signal does not have to contain the truth. So the players can deceive each other. Those possibilities are not so important in repeated games. In such games players get to know each other better. As a result they can predict the value of the signal. If players are taking part in an infinite game, where the number of moves is hard to define, there are no incentives to cheat. The players find it better to cooperate.

In this article we examine decision making process on the base of available signal in a game under uncertainty. Thus, the main aim of this article is to show, how to model a problem of transforming a signal into information in a way that shows the causes and results of it. In this article we use a version of the beer-quiche game of Kreps. We also show the possible development of this model. Theory of games enables us to acquire a better understanding of the decision making process in such a situation.

immediately by players, are the games of complete and imperfect information (Rasmusen, 2006).

The first stage of research task realization is to determine proxies for the level of asymmetric information. Then we discuss and compare models and methods based on those proxies. The main paper's thesis is that setting appropriate proxies enables analysts to determine the level of asymmetric information and the way it influences decision making process.

Signalling game

Applying the theory of asymmetric information into the theory of the firm it is mainly claimed that it is the manager who has an informational advantage towards the others, for example investors, banks, customers. Private information that she possesses concerns for example the value of shares in the company or its investment possibilities. In those cases information in possession of uninformed entities⁵ is also imperfect and incomplete.

In this article we analyze signaling i.e. the situation in which informed entities send signals to others. A signal can convey information, which makes uninformed entities determine new decisions and new strategies. We examine decision making process on the base of available signal in a game under uncertainty.

Signalling games finds its application in economics. The first application of signaling into economic problems was Michael Spence's model. It is a model of job market signaling describing a game where employees have a certain level of ability that the employer does not know. However employee can signal the level of his ability by presenting his diplomas i.e. his level of education. Employers observe the potential workers' diplomas (education), but they have no information about their real abilities. However, there is an assumption that can be made: only for the workers with a high level of abilities the benefits of education are greater than the costs. As a result only workers with a high ability can get a higher level of education.

To explain making decision on the base of signalling in general, we should describe the situation in terms of game rules. The rules of the game consist of players, actions, payoffs, and information. Depending on information that is available to each player, the strategies for each player should also be determined. The equilibrium is the combination of strategies chosen by each player.

⁵ It would be more precise to define these entities (or players) as partly informed. However in the theory of information entities are divided into informed ones and uninformed ones.

Table 1. Game theory's symbols in a game of signaling

symbol	meaning
t	type
T	set of types
m	message
M	set of messages
a	action
A	set of actions
S	sender
H, L	high, low
R	receiver
u	utility
v	-

Source: own compilation.

Signalling game is a dynamic game with two players: the sender (S) and the receiver (R). Sender is in the possession of some private information, which enables him to indicate the type $t \in T$ of which he is. So he signals it by sending the message $m \in M$ to an uninformed player i.e. to the receiver (Table 1). As a result the game of signaling consists of:

- information set for informed player (the sender), who sends signals based on his private information;
- information set for uninformed player (the receiver), who receives signals; this set includes actions $a \in A$ depending on the signals, which enables him to make decisions based on information derived from the signal;
- utility function of the sender $u(t, m, a)$;
- utility function of the receiver $v(t, m, a)$.

This article presents problems of asymmetric information and signalling using game theory on the base of beer-quake game of Kreps (I.-K. Cho, D. Kreps, 1987) and the version of K. Binmore (Binmore, 1992). In the original game of Kreps are two players. The first moves nature, which chooses the type of the first player (agent) as a *wimp* or *surly type* and reveals this information only to him, but not to the second player. With probability p the first

player is a surly one, and with probability $(p-1)$ he is a wimp. He signals his type by either choosing quiche or a beer. Second player observes the signal chosen by the first player (sender) and decides whether to be nice to him, because he is a surly type or to duel him because he is a wimp type.

In the stylization of Kreps's game, in this article, there are also two players and the Nature, which moves first. The players (agents) are the Borrower and the Bank. The Borrower is the informed one, so she is the sender and the Bank is the uninformed one, so he is the receiver. In economic literature it is common practice to call informed entity as she, and the uninformed one as he (Ritzberger, 2002).

The Bank has to make a decision whether to grant investment credit to a Borrower or not. The Nature can be given for example as an economic situation. The Nature moves first and chooses the type of the Borrower either as t_H or t_L , where:

- t_H is a firm of the high profits (S_H) with probability p ,
- t_L is the firm of the low profits (S_L) with probability $(p-1)$.

The performance of the firm is revealed only to Borrower, but not to the Bank. Then the Borrower chooses whether to present the real profit and loss statement or to improve it by securitization or creative accounting. As a result the profit and loss statement serves as a signal sent to the Bank. The next move is for the Bank. There are two assumptions that have to be made. The first is that the Borrower was not previously a client of the Bank, so the Bank can not derive data of his real situation from previous profit and loss statements. In a situation of a long-run relation between them, it would be easier for the Bank to determine the type of Borrower as t_H or t_L . That fulfils the assumption of asymmetric information between the players. The second one is that players are risk-neutral, so the payoffs can be represented by the profits of players, regardless of the utility functions.

According to the game of signaling the certain payoff is the function of the sender company's type, the signal sent by sender and the decision made by receiver (the Bank). For the Bank it is profitable to give a credit to a company of high profits S_H (payoff 5) or decline a credit application of a low-profit company S_L , which means that in this situation the Bank can find some better allocation for its resources (payoff 5). Borrower's payoffs depend on the fact if he was granted a credit. Each of Borrower's type that was granted a credit has a payoff of 5. Her payoff increases by 2, if she has presented authentic profit and loss statement, because she doesn't have to pay the cost of improving it by securitization or creative accounting.

Table 2. Normal form matrix for the low-profit Borrower

Bank		
Profit and loss statement of S_L	Decline a credit application	Accept a credit application
high-profit	0 5	5 0
low-profit	2 5	7 0

Source: own compilation.

Above-mentioned situation creates not only the condition of asymmetric information with signalling, but also the condition of incomplete information. Before ascribing probabilities to each type of player, it would be the game with uncertain information. Such a game is usually represented in a normal-form by a payoff matrix. The matrix shows the players' possible actions and payoffs: for the low-profit Borrower S_L (Table 2) or for a high-profit Borrower S_H (Table 3).

Table 3. Normal form matrix for the high-profit Borrower

Bank		
Profit and loss statement of S_H	Decline a credit application	Accept a credit application
high-profit	2 0	7 5
low-profit	0 0	5 5

Source: own compilation.

To sum up, the possible strategies of the Bank (not knowing the authentic performance of the Borrower) are

- S1: If Borrower presents high profit (H), decline the credit application; if Borrower presents low profit (L), decline the credit application.
- S2: If Borrower presents H, decline the credit application; if Borrower presents L, accept the credit application.
- S3: If Borrower presents H, accept the credit application; if Borrower presents L, decline the credit application.

- S4: If Borrower presents H, accept the credit application; if Borrower presents L, accept the credit application.

According to these strategies the game can be represent by a strategic-form matrices (Table 4, Table 5).

Table 4. The strategic form matrix for the low-profit Borrower

Bank Profit and loss statement of S_L	If Borrower presents H, decline the cre- dit application; if Borrower presents L, de- cline the credit application	If Borrower presents H, decline the cre- dit application; if Borrower presents L, ac- cept the credit application	If Borrower presents H, accept the cre- dit application; if Borrower presents L, de- cline the credit application	If Borrower presents H, accept the cre- dit application; if Borrower presents L, de- cline the credit application
	high-profit	0 5	0 5	5 0
low-profit	2 5	7 0	2 5	7 0

Source: own compilation.

Table 5. The strategic form matrix for the high-profit Borrower

Bank Profit and loss statement of S_L	If Borrower presents H, decline the cre- dit application; if Borrower presents L, de- cline the credit application	If Borrower presents H, decline the cre- dit application; if Borrower presents L, ac- cept the credit application	If Borrower presents H, accept the cre- dit application; if Borrower presents L, de- cline the credit application	If Borrower presents H, accept the cre- dit application; if Borrower presents L, de- cline the credit application
	high-profit	2 0	2 0	7 5
low-profit	0 0	5 5	0 0	5 5

Source: own compilation.

The solution of this problem is based on the methodology proposed by J. Harsányi (Harsányi, 1967-8). Bank makes the assumption, that with probability p the Borrower is a high-profit company, and with probability $(p-1)$ she is a low-profit company. As it was said before, probabilities can depend for instance on the Bank (or more precisely: bank officer's)

previous experience. As a result the matrix depending on probabilities can be determined.

Taking under consideration the situation, when a Borrower has presented high-profit profit and loss statement and the Bank declines the credit application (the first row and the first column in Tables 2 and 3), Bank's payoffs are: 5 in case of the low-profit company and 0 in case of the high-profit company. To fill in the matrix, the probabilities should be taken into account. The payoff for the Bank associated with the low-profit company equals $(p-1) \cdot 5$ and associated with the high-profit company equals $p \cdot 0$. As a result Bank's payoff in this situation is $(p-1) \cdot 5 + p \cdot 0 = 5p - 5$ (Table 6).

Determining the payoffs for each situation allows constructing the normal form matrix of Bank's payoffs (Table 6) or the strategic form matrix of Bank's payoffs (Table 7) depending on probability p .

Table 6. Normal form matrix of Bank payoffs depending on probability

Bank	Decline a credit application	Accept a credit application
Profit and loss Statement of S_L		
high-profit	$5 - 5p$	$5p$
low-profit	$5 - 5p$	$5p$

Source: own compilation.

Table 7. Strategic form matrix of Bank payoffs depending on probability

Bank	If Borrower presents H, decline the credit application; if Borrower presents L, decline the credit application	If Borrower presents H, decline the credit application; if Borrower presents L, accept the credit application	If Borrower presents H, accept the credit application; if Borrower presents L, decline the credit application	If Borrower presents H, accept the credit application; if Borrower presents L, decline the credit application
Profit and loss Statement of S_L				
high-profit	$5 - 5p$	$5 - 5p$	$5p$	$5p$
low-profit	$5 - 5p$	$5p$	$5 - 5p$	$5p$

Source: own compilation.

In this game of incomplete and asymmetric information there are several solutions, that can be achieved, in other words there are several types of equilibrium. Solving the problem is based on the concept of perfect Bayesian Nash equilibrium (PBE). Nash equilibrium is a given profile of strategies such that no player has an incentive to unilaterally deviate from his equilibrium strategy. So the strategies creating equilibrium are the best responses for the strategies of all other players.

A perfect Bayesian Nash equilibrium is a refinement of Nash equilibrium for sequential games with incomplete information. In those games players form a complete system of beliefs about the opponents' types at each node of the game that can be reached. It is because a type of a player determines the decisions and actions that can be undertaken by him. So the strategies along the equilibrium path are based on the beliefs because of incomplete information. The system of beliefs can be updated according to Bayes' rule, so updating the probabilities occurs as conditional probability conception. As a result a perfect Bayesian Nash equilibrium is a unilaterally unimprovable strategy profile and a set of beliefs updated according to Bayes' rule:

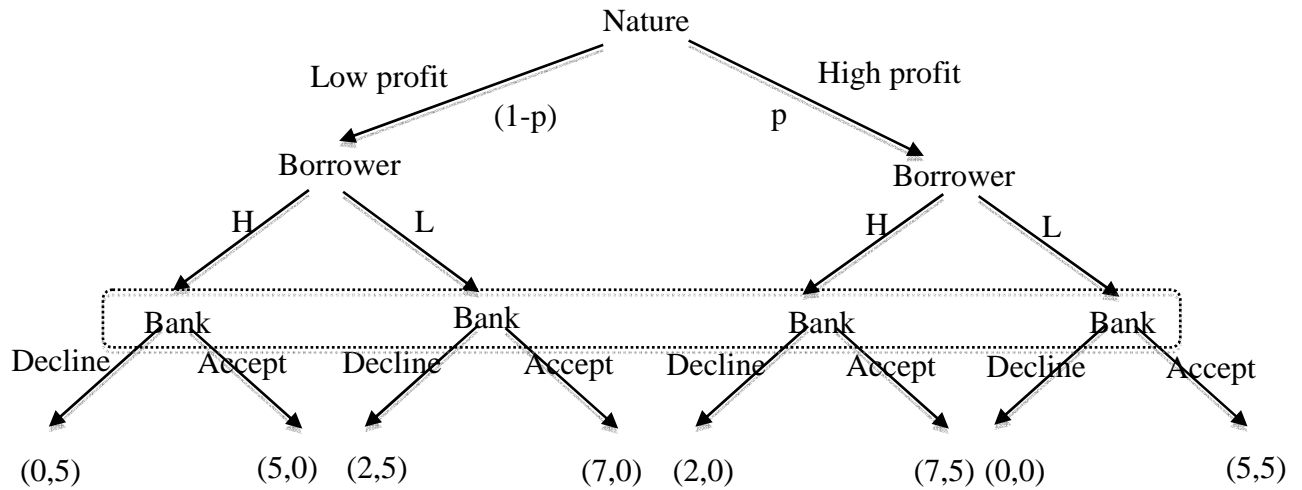
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B)}$$

That is the reason, why the games of incomplete information, which solution can be found by introducing conditional probabilities, are called Bayesian games, and the entities in those games are called Bayesian players.

If each type chooses the same signal, the equilibrium would be so-called pooling perfect Bayesian Nash equilibrium (pooling PBE), which results in the fact that observed signal reveals no additional information.

If each type chooses a different signal, it would be separating PBE, where the signal precisely identifies the borrower's type. However, in reality it will not always be profitable for low-profit borrower to send the signal containing the truth. So bank officers and analytics have to take it under account and predict the probability with which they can expect that the message is true. Though it is even easier to determine the probability with which the sender could be high-profit company S_H .

When one type of Borrower mixes between two signals, which can be also chosen by the other type of Borrower, some information about her type can be inferred from the signal. In such a case the equilibrium is called semiseparating PBE.



Payoffs for (Borrower, Bank).

Figure 1. Extensive form of a game

Source: own compilation.

For sequential games, it is more intelligible to present the problem in an extensive form, known also as a decision tree. An extensive form of this game is given in Figure 1. It is assumed, that the first choice is made by Nature. There are no payoffs for a nature, but she does have the first node under control. After the Nature's choice, the Borrower knows the Nature's decision, but the Bank does not. The Borrower and the Bank decide simultaneously. That is why Bank's nodes are joined by a dashed line indicating the information set.

In the extensive form of this game there are two possible equilibrium outcomes for different levels of probability (Cho, Kreps, 1987). To solve the game strategic form matrix can be analyzed (Table 8 and 9). Bank's strategies have been discussed above. Borrower can compose four possible strategies:

- Always H: present high profits regardless of the choice of Nature.
- L-L/H-H: If Nature chooses her type as low-profit, show low profit (L); if Nature chooses her type as high-profit; show real profit (H).
- L-H/H-L: If Nature chooses her type as low-profit, show high profit (H); if Nature chooses her type as high-profit; show low profit (L).
- Always L: present low profit regardless of the choice of Nature.

For $p < \frac{1}{2}$, the equilibrium is quite obvious (Table 8). The bank strategy is: if Borrower chooses to present low profits, the Bank declines the credit application, but if Borrower chooses to present high profits, the Bank is indifferent between accepting and declining the credit application (*semi-separating PBE*).

Table 8. A strategic form game corresponding extensive form game from Figure 1 and the solution for $p < \frac{1}{2}$

Bank's strategies	If Borrower presents H, decline the credit application;		If Borrower presents H, decline the credit application;		If Borrower presents H, accept the credit application;		If Borrower presents H, accept the credit application;	
Borrower's strategies	if Borrower presents L, decline the credit application		if Borrower presents L, accept the credit application		if Borrower presents L, decline the credit application		if Borrower presents L, decline the credit application	
Always H	$2p$	$5-5p$	$2p$	$5-5p$	$5+2p$	$5p$	$5+2p$	$5p$
L-L/H-H	2	$5-5p$	$7-5p$	0	$2+5p$	5	7	$5p$
L-H/H-L	0	$5-5p$	$5p$	5	$5-5p$	0	0	$5p$
Always L	$2-2p$	$5-5p$	$7-2p$	$5p$	$2-2p$	$5-5p$	$7-2p$	$5p$

Source: own compilation.

For $p > \frac{1}{2}$, the Bank gives a credit, regardless of the presented profit and loss statement. In this situation we can find the solution using dominance arguments. When $p > \frac{1}{2}$ then $5-5p \leq 5p$, and strategy 'always accept the credit application' dominates strategy 'always decline the credit application' (Table 6).

For $p > \frac{1}{2}$ there is also one imperfect equilibrium, a paradox: the Bank declines credit application, if the Borrower presents high profits, and borrowers present low profits.

This equilibrium can be found in a strategic form corresponding to extensive form of the game (Table 9).

Table 9. A strategic form game corresponding extensive form game from Figure 1 and the solution for $p > \frac{1}{2}$

Bank's strategies Borrower's strategies	If Borrower presents H, decline the credit application; if Borrower presents L, decline the credit application	If Borrower presents H, decline the credit application; if Borrower presents L, accept the credit application	If Borrower presents H, accept the credit application; if Borrower presents L, decline the credit application	If Borrower presents H, accept the credit application; if Borrower presents L, decline the credit application
Always H	$2p$ $5-5p$	$2p$ $5-5p$	$5+2p$ $5p$	$5+2p$ $5p$
L-L/H-H	2 $5-5p$	$7-5p$ 0	$2+5p$ 5	7 $5p$
L-H/H-L	0 $5-5p$	$5p$ 5	$5-5p$ 0	0 $5p$
Always L	$2-2p$ $5-5p$	$7-2p$ $5p$	$2-2p$ $5-5p$	$7-2p$ $5p$

Source: own compilation.

The payoffs are computed as expected values. For example in upper-left quadrant Borrower gets a payoff of $2p$ and Bank gets $5-5p$. When the Nature's decision is L, Borrower presents H and Bank refuses the credit. That gives 0 to Borrower and 5 to Bank. When Nature's decision is H, Borrower presents H and Bank declines the credit application. That gives 2 for Borrower and 0 for Bank (Figure 1). Since the first possibility has probability $(1-p)$ and the second p , the expected payoffs from this pair of strategies are $(1-p) \cdot 0 + p \cdot 2 = 2p$ for Borrower and $(1-p) \cdot 5 + p \cdot 0 = 5-5p$ for Bank. The other payoffs are computed similarly.

When players get more and more information and experience, their beliefs change. Players modify their beliefs and their expectations, by describing new values of probability to specific situations. To compute conditional probability the Bayes' theorem is used.

Acquiring access to the credible and complete information is a wide problem and it concerns many aspects of everyday life. When the characteristics and analysis of researching problem are difficult because of uncertainty, it is possible to find the solution using methods of game theory. It allows the researcher to use randomization and transform problem in an incomplete information game. As a result it becomes intelligible and solvable. For the repeat games this works like a learning effect.

Summary

An example introduced in this article shows possibilities of modeling problems connected with asymmetric information and signalling. Presented problem of the creative accounting and securitization becomes more and more popular in a economic literature. These are the means of creation of the layout of companies' balance and profit and loss statement - documents, that present capital structure of companies, that are the picture of company for the potential investors and partners. However, they are made in the asymmetrical information. Moreover they became the means of conscious introducing asymmetrical information for gaining additional advantages.

There is a wide range of applications for an analysis of signalling. Problems of asymmetrical and incomplete information can be found not only in banking, but also in problems of employment, accounting, relation between managers and shareholders. It concerns situations in which it is impossible to increase directly the flow of information.

The methods of game theory make the problem more intelligible. Theory of games enables researcher to find equilibrium in dynamic problem in the contests of Bayes' theorem and learning effect.

Comprehensive check ???

1. In which fields of the company activities can you find signaling?
2. Suggest other problems, that can be solved using this model.
3. What is the role of the nature in models of signaling based on the game theory?
4. If the assumption, that the Borrower was not previously a client of the Bank, was overruled, how it would affect the model?

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